

M.Sc. (Maths.) Semester - 4 (CBCS) Examination
March/April - 2025 (NEW COURSE)
Number Theory -2 (Core)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

- Que-1(A) Answer any two out of three (10)
- (1) Explain the role of Farey fractions in rational approximations.
 - (2) Explain how to find median of two fractions in a Farey sequence.
 - (3) Show that no two fractions in a Farey sequence are equal.
- Que-1(B) Answer any one out of two (04)
- (1) Define Farey sequence of order 5.
 - (2) Find the continued fraction expansion of $\frac{7}{4}$.
- Que-2(A) Answer any two out of three (10)
- (1) Explain the process of converting a rational number into a continued fraction.
 - (2) What is a simple continued fraction? Give an example.
 - (3) Compute the first three convergence of $[2; 3, 1]$.
- Que-2(B) Answer any one out of two (04)
- (1) Define finite continued fractions and give an example.
 - (2) Find the Farey sequence of order 3.
- Que-3(A) Answer any two out of three (10)
- (1) Explain how to determine periodicity in continued fractions.
 - (2) Solve the Pell's equation for small values of $x^2 - 3y^2 = 1$.
 - (3) Find the continued fraction expansion of $\sqrt{7}$.
- Que-3(B) Answer any one out of two (04)
- (1) Define purely periodic continued fractions.
 - (2) Describe the relationship between periodic continued fractions and quadratic irrationals.
- Que-4(A) Answer any two out of three (10)
- (1) Derive formula to generate Pythagorean triplets.
 - (2) Solve the Diophantine equation $2x + 3y = 5$ in integers.
 - (3) Find integer solutions for the equation $5x + 7y = 1$.
- Que-4(B) Answer any one out of two (04)
- (1) Solve the equation $x^2 + y^2 = 50$ in integers.
 - (2) Find a Pythagorean triplet where one element is 12.
- Que-5(A) Answer any two out of three (10)
- (1) Prove that $x^2 - dy^2 = 1$ has infinitely many solutions for any non-square integer $d > 0$.
 - (2) Discuss a special case of a Diophantine equation and its solution.
 - (3) Discuss the role of Pell's equation in finding integer solutions.
- Que-5(B) Answer any one out of two (04)
- (1) Explain the concept of rational approximations using continued fractions.
 - (2) Prove that if $\frac{a}{b}$ and $\frac{c}{d}$ are two consecutive terms in a Farey sequence, then $bc - ad = 1$.
